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# Defining Object Type Using MDP Homomorphisms

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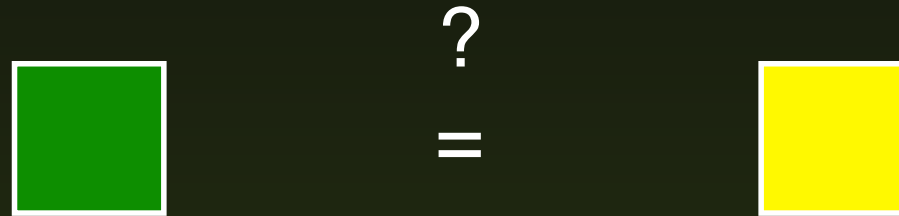
# Outline

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- Introduction: Object Type
- CMP Homomorphisms
- Object Homomorphisms
- Object Options
- Subtypes
- Discussion

# Modeling Objects

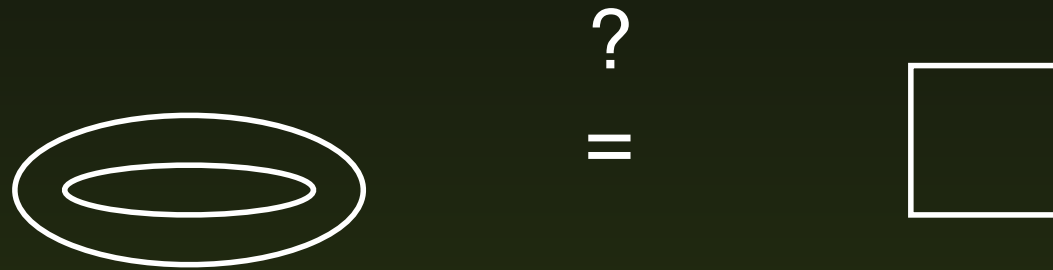
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- Are green blocks the same as yellow blocks?
- Could the same policy be used to move both?

# Modeling Objects

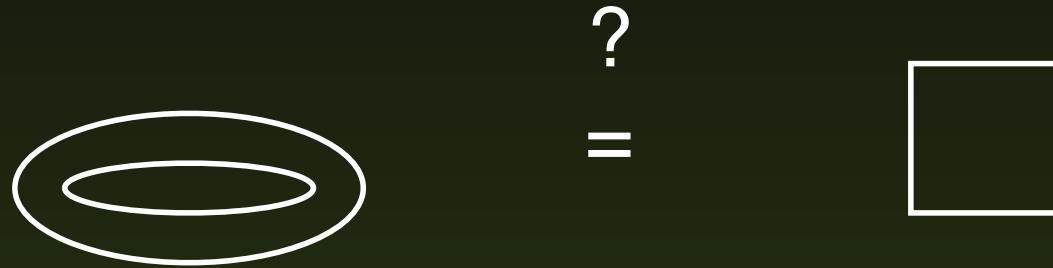
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- Is a block the same as a plate?

# Modeling Objects

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- Is a block the same as a plate?
- Can they be stacked the same way?

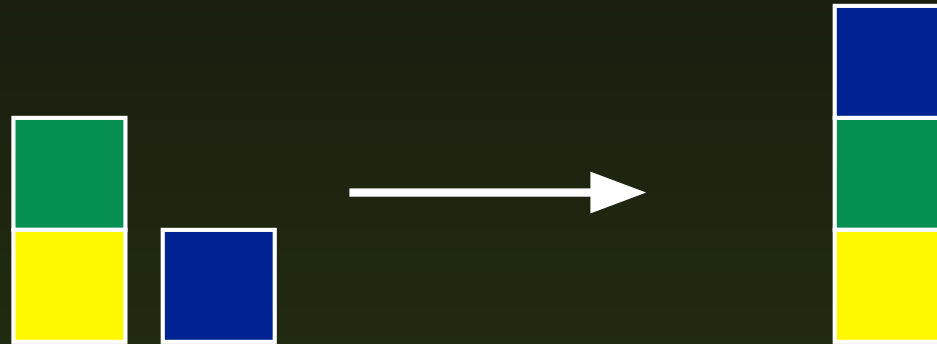
# Related Work

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- Givan, R., Dean, T., & Greig, M. *Equivalence Notions and Model Minimization in Markov Decision Processes*. Artificial Intelligence, 2003
  - stochastic bisimulation
- Ravindran, B. & Barto, A. G. *SMDP Homomorphisms: An Algebraic Approach to Abstraction in Semi Markov Decision Processes*. IJCAI-03
  - MDP Homomorphisms
- CMP Homomorphisms (Wolfe, Barto, AAAI 2006)
  - If you are going to bother to build a model, use it for multiple tasks

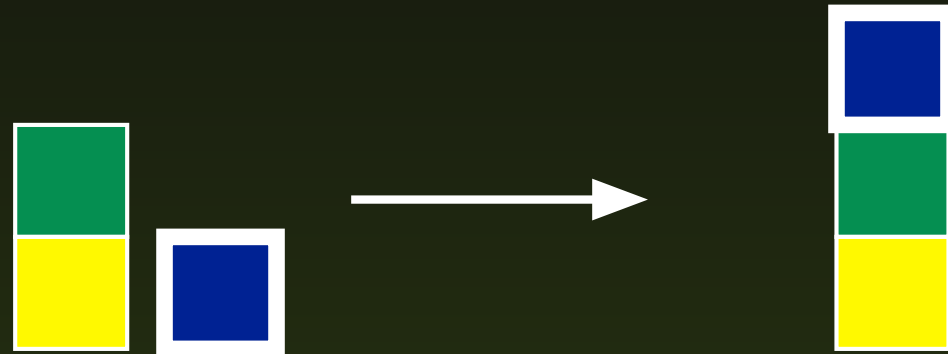
# Controlled Markov Processes

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- Controlled Markov Process:  $(S, A, T)$
- $S$ : State set,  $A$ : Action set,  $T : S \times A \times S \rightarrow [0, 1]$

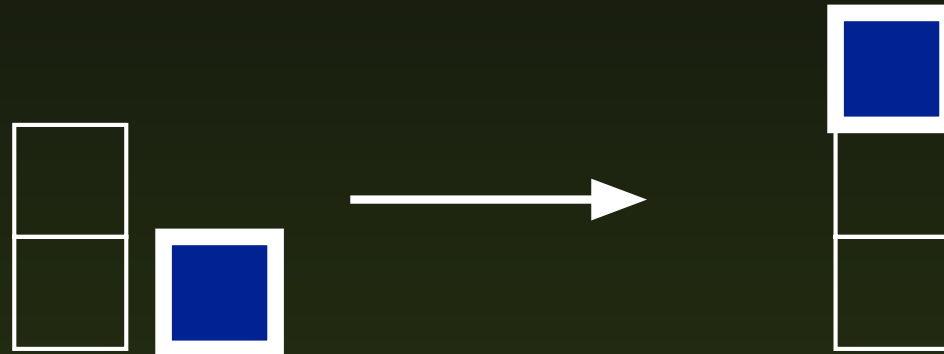
# Controlled Markov Processes



- Controlled Markov Process:  $(S, A, T)$
- $S$ : State set,  $A$ : Action set,  $T : S \times A \times S \rightarrow [0, 1]$
- Add output variable:  $(S, A, T, y)$
- $y : S \rightarrow Y$



# CMP Homomorphisms



- Model which predicts one specific output variable
- Transitions occur between abstract states
- Can build policies for supported reward functions

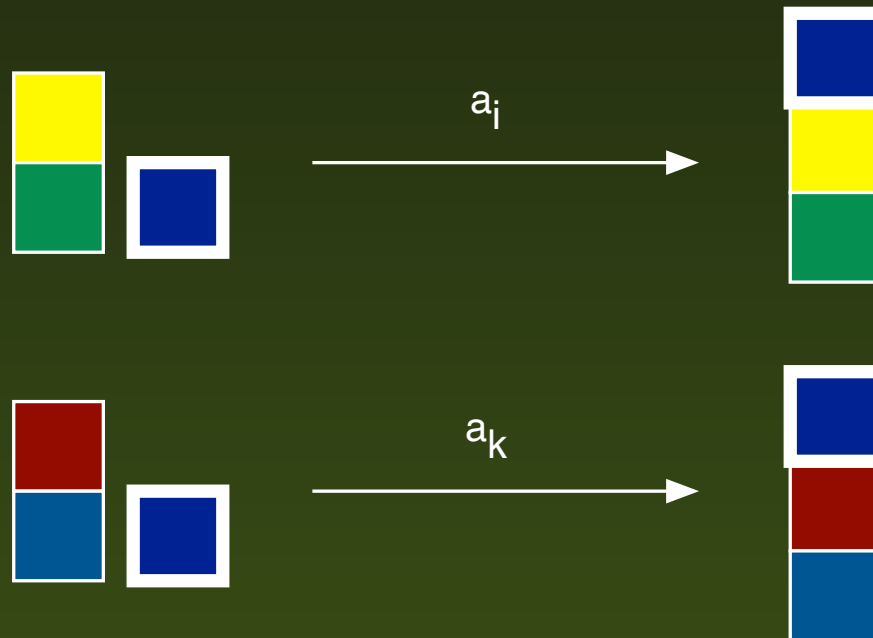
$$r \circ y$$

# CMP Homomorphisms

- Partition of state and action spaces, with constraints:

$$y(f(s), g_s(a)) = y(s, a)$$

$$T(f(s_i), g_s(a), f(s_j)) = \sum_{s_k | f(s_j) = f(s_k)} T(s_i, a, s_k)$$

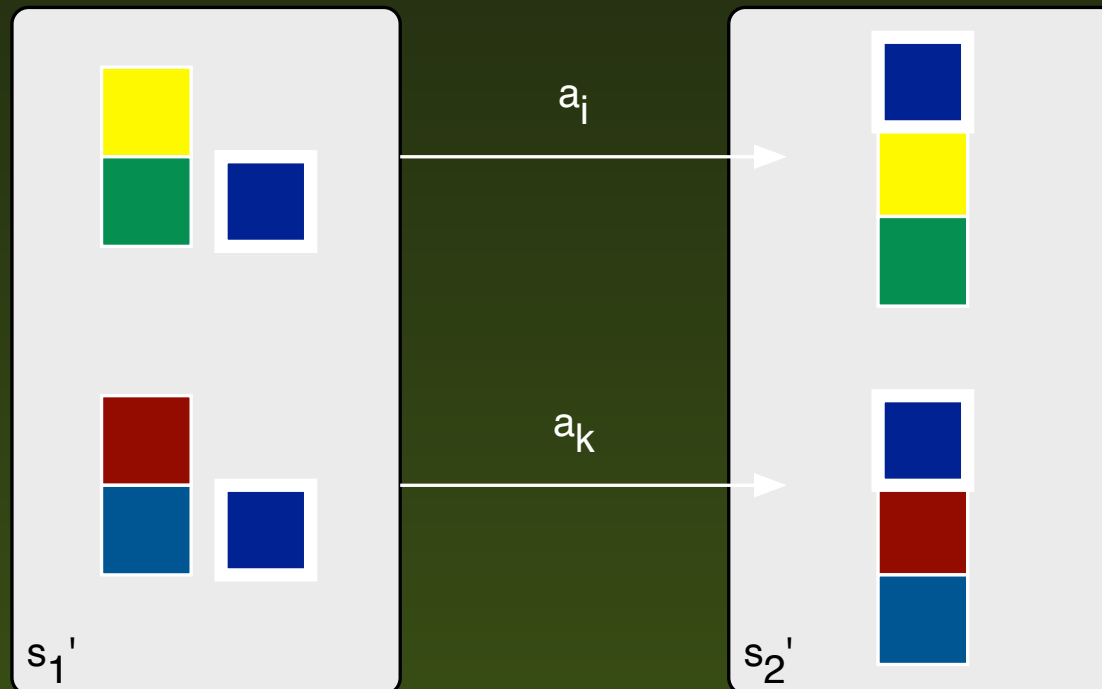


# CMP Homomorphisms

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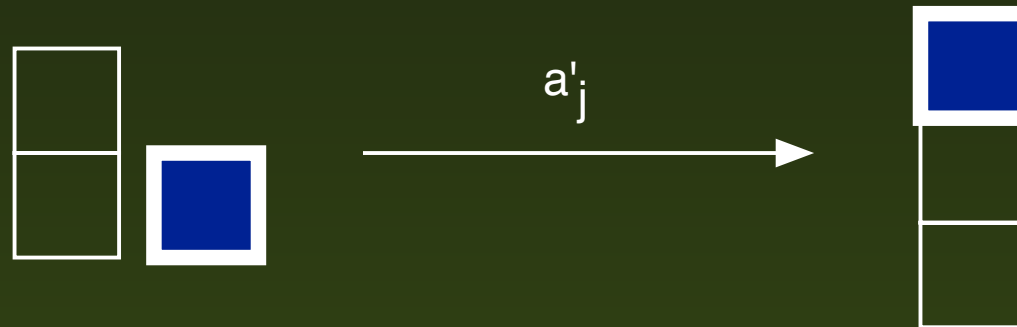


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# Object CMPs

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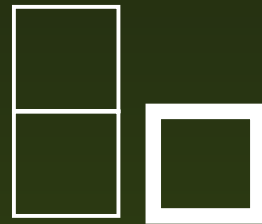
- Output is  $z \circ w_o$  where  $w_o$  singles out object  $o$ , and  $z$  singles out a feature
- What if multiple objects have the same model for  $z$ ?



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# Generalization

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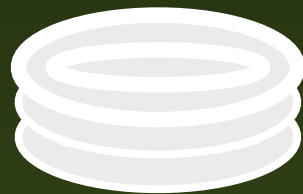
- Plates, blocks  $\in$  stackable objects type
- Only have to be the same with respect to the output variable



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# Lifting Policies

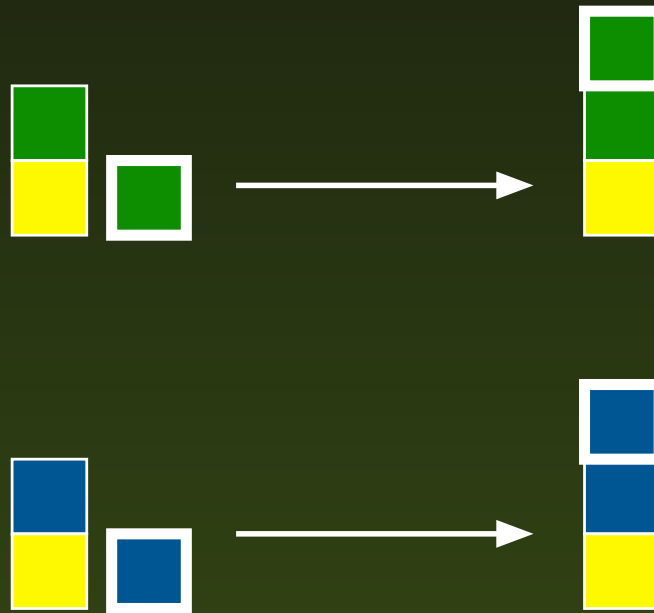
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- Policy specifies action in abstract model



# Lifting Policies

- Policy specifies action in abstract model
- Reverse mapping to find the corresponding action in the CMP



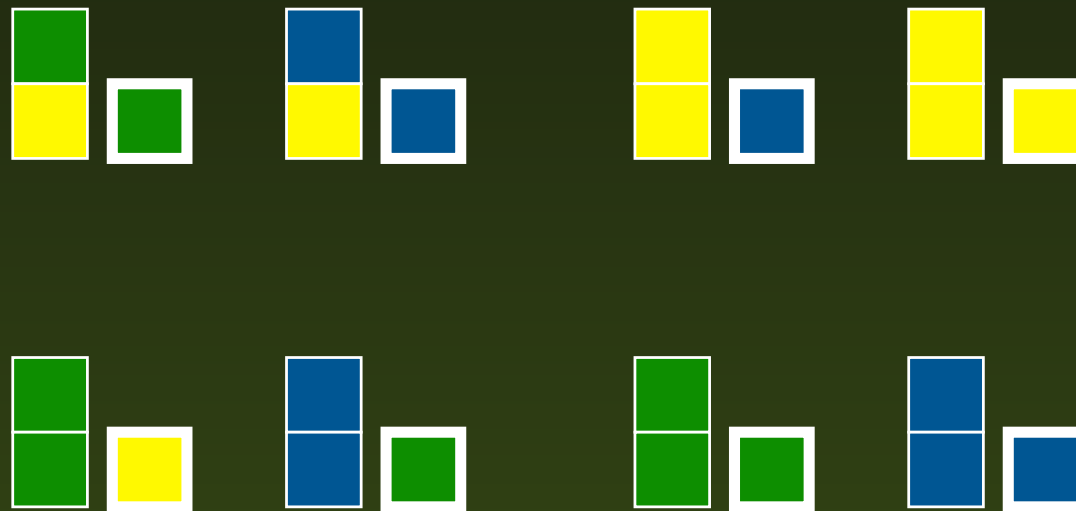
# Object Options

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- Subgoal option:
  - reward function  $r$
  - termination function  $\beta$
- Object option: both are function of  $z$
- Only need to find policies for types, not specific objects

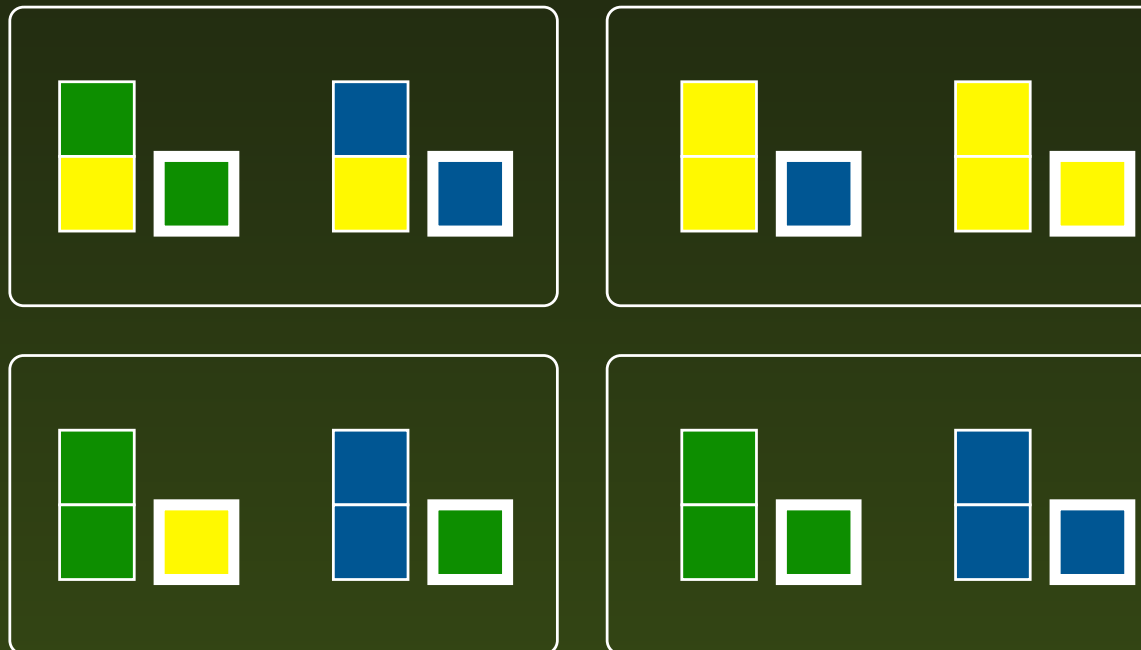
# Object Type: Subtypes

- What if all blue and green blocks stick to blocks of the same color, but yellow do not?
- Sample states:



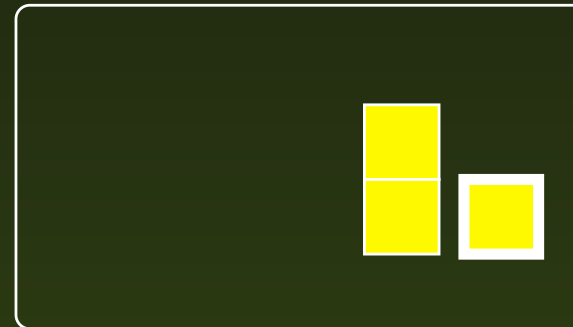
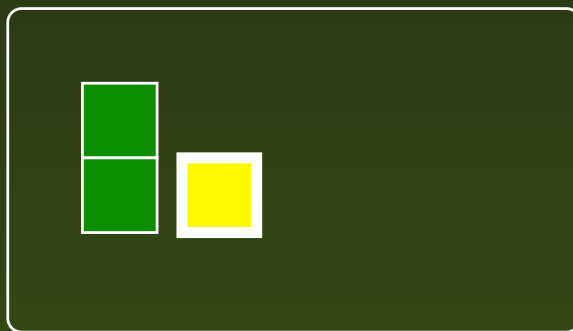
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# Object CMPs

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- Equivalence criteria:
  - $\forall$  CMPs  $M_k$
  - $h_i$  the reduction of  $M_k, z \circ w_{o_i}$
  - $\exists h_j, M_l, h_j$  a reduction of  $M_l, z \circ w_{o_j}$
  - Such that  $h_i(M_k, z \circ w_{o_i}) = h_j(M_l, z \circ w_{o_j})$
  - Then  $o_j \preceq o_i$  under the output  $z$



# Discussion

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- View environment from point of view of a single object
  - could be another agent
- Alternate method: add "pointer" to state space
  - one large model over all types
- HM framework does not generalize to more objects
  - Can't use reduction for 3 blocks to learn about 4
  - Find the relations which will generalize from examples of reductions
  - Build a generic reduction